Information and Coding Theory

ENEE 5304

Problem Set 3

Optimum Receivers and Channel Capacity

1. Two random variables X and Y are distributed according to

$$f_{XY}(x,y)=\begin{cases}K(x+y), & 0\le x,y\le 1\\ 0, & otherwise\end{cases}$$

a) Find K.

b) Find the probability $P(X+Y>1)$.

2. A noise process has a power-spectral density given by

$$G_n(f)=\begin{cases}10^{-8}\left(1-\dfrac{|f|}{10^8}\right), & |f|<10^8\\ \\ 0, & |f|>10^8\end{cases}.$$

This noise is passed through an ideal bandpass filter with a bandwidth of 2 MHz centered at 50 MHz. Find the power content of the output process.

3. Find the differential entropy of the continuous random variable X with an exponential random variable with parameter $\lambda>0$

$$f_X(x)=\begin{cases}\dfrac{1}{\lambda}\exp(-x/\lambda), & x>0\\ 0, & otherwise\end{cases}.$$

4. Calculate the information rate in bits/sec of a telegraph source having two symbols, dot and dash. The dot duration is 0.2 s, the dash is twice as long as the dot and half as probable.

5. Consider discrete memoryless channel with the transition matrix

$$P_{ij}=\begin{bmatrix}1 & 0\\ 0.5 & 0.5\end{bmatrix}.$$

Find the capacity of the channel.

6. Find capacity of an additive Gaussian white noise channel with a bandwidth of 1 MHz, power of 10 W, noise power-spectral density of $\frac{N_0}{2} = 10^{-9}$ W/Hz.

7. Consider a binary symmetric channel characterized by the transition probability *p*. Plot the mutual information of the channel as a function of P1, the *a priori* probability of symbol 1 at the channel input; do your calculations for the transition probability *p*=0, 0.1, 0.2, 0.3, 0.5.

8. The binary orthogonal frequency shift keying (FSK) signaling scheme employs the following two equally probable signals $s_1(t)$ and $s_2(t)$ to represent binary logic 0 and 1 respectively over a channel corrupted by AWGN with N_0 = 0.001 W/Hz:

$$s_1(t) = 4\cos(2\pi f_1 t), \quad 0 \le t \le T_b$$
$$s_2(t) = 4\cos(2\pi f_2 t) \quad 0 \le t \le T_b,$$

a. If the bit error probability is not to exceed 10^{-4}, find the maximum allowable data rate R_b in bits per second.
b. Sketch the optimum demodulator

9. Let X have a density and let H(X) denote the differential entropy. Show that for any a > 0 we have , H(aX) = H(X) + log a.
10. The joint probability mass function of two random variables X and Y is shown in the table below.

		Y	
		2	3
X	0	0.45	0.12
	1	0.15	0.28

a. Find H(X)
b. Find I(X; Y)